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Fading Margin Analysis for Asynchronous CDMA Systems

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In this paper, a detailed theoretical analysis of fading margin in an asynchronous code division multiple access (A-CDMA) system is discussed. Rayleigh and Rician frequency-selective slowly fading channels are considered. Probability distribution and density functions of the probability of error are derived for Rayleigh and Rician fading channels. The fluctuations in the channel capacity are proved to be directly proportional to the signal-to-noise ratio (SNR) variations. Fading margin is calculated for both Rayleigh and Rician fading channels as a function of the probability of error specification and the probability of unsatisfactory operation.

KEY WORDS: Fading margin; channel capacity; error probability distribution and density functions; sensitivity.

1. INTRODUCTION

Direct sequence CDMA systems incorporate signals which have a transmission bandwidth which is several orders greater than the minimum modulation bandwidth. All users in CDMA systems employ the same bandwidth for transmission. Since multiple users simultaneously occupy the available bandwidth, the users interfere with one another significantly. Due to the nature of the propagation mechanism, the signal received from a base station from a user terminal close to the base station will be stronger than the signal received from another terminal located at the cell boundary [1]. Hence, the users close to the base station dominate those which are far from the base station. This effect, called the near–far effect can be reduced by attempting to achieve a constant received mean power of each user.

Capacity can be increased by having the signals arrive at the base station with the same mean power [1]. Thus, transmitter power control becomes a very important factor in determining the capacity of a DS-CDMA system. The number of CDMA channels in a network depends on the level of interference that can be tolerated in the system. Thus, desired system quality plays an important role in determining the overall capacity of the system.

In [2] power is increased or decreased based on the current value of the carrier-to-interference ratio. The capacity and quality of communications in cellular mobile radio systems is increased by controlling the base station transmitter powers. The idea of enhancing the throughput using adaptive power control for direct-sequence spread-spectrum packet radio networks is investigated in [3].

In [4] and [5], for mobiles which are in soft hand-off, a lower fading margin is obtained to provide desired coverage as compared to mobiles which are in hard hand-off. This is because a mobile which is in soft hand-off maintains simultaneous radio links with multiple base stations which enables it to make use of the best quality leg most of the time. In [6], the statistical variation of both the received signal and interference power are considered to optimally allocate power, subject to constraints on the probability of fading-induced outage (which occurs when the signal-to-interference ratio (SIR) falls below a threshold SIR) for each transmitter/receiver pair.

In [7], the problem considered is to support downlink non-real time data services, where in addition to power control, there is also the possibility

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of controlling the interference by means of transmission scheduling. By having the users transmit in a one-by-one fashion within each cell, energy is saved and system capacity is increased.

In [8], the interest is in developing a stochastic algorithm for the problem of optimally allocating power to deal with noise corrupted measurements that serve as inputs to the problem. In [9], SIR based power control algorithms are used to increase capacity and improve quality of service in a CDMA system.

The idea of supporting different quality of service (QoS) requirements for different traffic in an interference-limited cellular CDMA environment is studied in [10] and [11]. In [12], it is argued that the distribution of the estimation error in the average power under combined Rayleigh fading and shadowing follows a log-normal distribution. This model has many applications in cellular systems.

Yates in [13] formulates a unified framework for uplink power control and its convergence. In [14], three basic interference management approaches, transmit power control, multiuser detection, and beamforming are combined to increase the uplink capacity of a CDMA system.

In this work, error probability distribution and density functions are derived to characterize the fading process in an asynchronous CDMA (A-CDMA) system. This work is different from the previous work [2–14] since it employs the distribution function of the probability of error, and its relation with the distribution function of the signal energy-to-noise ratio. These distribution functions are useful in computing the "fading margin" for Rayleigh and Rician fading channels.

Section 2 provides a description of the single-user, multiple-interferer A-CDMA system and derives expressions for received signal-to-noise ratio (SNR), and channel capacity in the A-CDMA system considered. Section 3 provides a detailed mathematical analysis of error probability distribution function, error probability density function, introduces the concept of "fading margin", and analyzes the theoretical results. Finally, Section 4 presents the conclusions.

2. SYSTEM DESCRIPTION

The block diagram shown in Figure 1 is that of a multiuser uplink (mobile users to base station) A-CDMA system [15]. The multiuser system can be considered as a single-user multiple-interferer system if the total effect of the $(U-1)$ interferers is approximately Gaussian. This approximation is valid if the interferer powers are non-negligible compared to the user power. The channel characteristics can be estimated in terms of an ensemble of receiver inputs [15]

$$r(t) = \sum_{k=0}^{U-1} h_k(t) * s_k(t-\delta_k) + n(t) = \sum_{k=0}^{U-1} x_k(t-\delta_k) + n(t) = x(t) + n(t), \quad (1)$$

where

$$s_k(t-\delta_k) = \sqrt{2P_k}\,a_k(t-\delta_k)b_k(t-\delta_k)\cos(\omega_c t + \Phi_k). \quad (2)$$

Here, * denotes the convolutional operator. The A-CDMA system specifications are as follows:

- P_k is the power of user k,
- a_k is the spreading sequence of user k,
- b_k is the data sequence of user k,
- δ_k is the delay of user k with respect to 0, and
- $h_k(t)$ is the impulse response of the multipath fading channel. It is modelled as a random

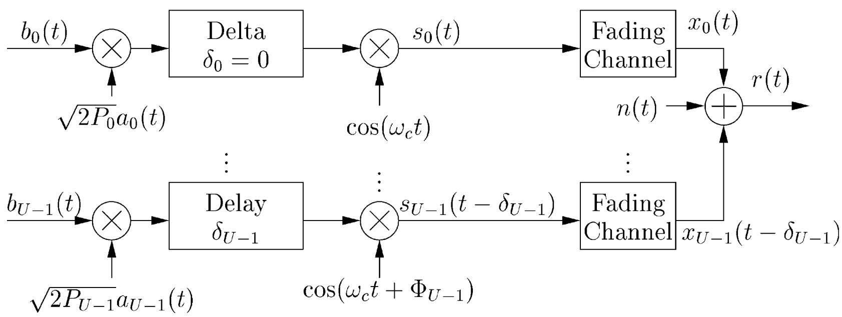

Fig. 1. Block diagram of an A-CDMA system with U users.

variable for user k that gives the envelope of the received signal.

In this paper, the multipath fading channels are either Rayleigh or Rician distributed. Let $A_k(t)$ be the received signal amplitude for user k and $\theta_k(t)$ be the phase shift of the kth user due to the fading channel. For a slowly fading channel, these values can be assumed to be a constant over a symbol duration T. In this paper, the fading is considered to be time-invariant. The transmitted signal from the kth user is $x_k(t-\delta_k) = A_k(t)\sqrt{2P_k}a_k(t-\delta_k)b_k(t-\delta_k)\cos(\omega_c t + \Phi_k + \theta_k(t))$. The user delays, δ_k, are due to the asynchronous system. There is also a second delay due to transmission. The transmission delay can be neglected if we assume perfect timing for user 0.

It is necessary to compute the received SNR of the concerned user before deciding to change the transmitter power of that particular user. The next subsection gives a detailed analysis on the received SNR measurements.

2.1. Computation of the Received Signal-to-Noise Ratio

It is reasonable to think of a short-time channel capacity $C(t)$. We assume that over a symbol duration, T, in which we estimate SNR(t), the received signal amplitude, $A_k(t)$, and the received signal phase, $\theta_k(t)$, change very little due to the slowly fading channel.

The short-term mean-square value of the received signal neglecting noise can be expressed as

$$\overline{x^2(t)}^T = \overline{\left(\sum_{k=0}^{U-1} x_k(t-\delta_k)\right)^2}^T = \overline{\left(\sum_{k=0}^{U-1} A_k(t)\sqrt{2P_k}a_k(t-\delta_k)b_k(t-\delta_k)\cos(\omega_c t + \Phi_k + \theta_k(t))\right)^2}^T, \tag{3}$$

where the bar T refers to a time-average over time-period T. Rewriting (3), we have

$$\begin{aligned}\overline{x^2(t)}^T &= \overline{\left[\sum_{k=0}^{U-1} A_k(t)\sqrt{2P_k}a_k(t-\delta_k)b_k(t-\delta_k)\cos(\omega_c t + \Phi_k + \theta_k(t))\right]^2}^T \\ &= \frac{1}{T}\int_0^T \left[\sum_{k=0}^{U-1} A_k(t)\sqrt{2P_k}a_k(t-\delta_k)b_k(t-\delta_k)\cos(\omega_c t + \Phi_k + \theta_k(t))\right]^2 dt \\ &= \frac{1}{T}\int_0^T \sum_{k=0}^{U-1}\sum_{l=0}^{U-1} A_k(t)A_l(t)\sqrt{P_k P_l}a_k(t-\delta_k)b_k(t-\delta_k)a_l(t-\delta_l)b_l(t-\delta_l) \\ &\quad \times [\cos(\Phi_k - \Phi_l + \theta_k(t) - \theta_l(t)) + \cos(2\omega_c t + \Phi_k + \Phi_l + \theta_k(t) + \theta_l(t))]\, dt.\end{aligned} \tag{4}$$

For $\omega_c \gg T^{-1}$, (4) can be approximated as

$$\begin{aligned}\overline{x^2(t)}^T \approx & \frac{1}{T}\sum_{k=0}^{U-1}\sum_{l=0}^{U-1} A_k(t)A_l(t)\sqrt{P_k P_l} \\ & \times \int_0^T a_k(t-\delta_k)a_l(t-\delta_l)b_k(t-\delta_k)b_l(t-\delta_l) \\ & \times \cos(\Phi_k - \Phi_l + \theta_k(t) - \theta_l(t))\, dt,\end{aligned} \tag{5}$$

since the received signal amplitudes, $A_k(t)$ and $A_l(t)$ of the kth and the lth users respectively, are assumed to be a constant over symbol duration T. If the aperiodic cross-correlations of the spreading sequences are small, (5) can be approximated as

$$\overline{x^2(t)}^T \approx \sum_{k=0}^{U-1} A_k^2(t)P_k. \tag{6}$$

The short-term mean-square value of noise can be expressed as

$$\overline{n^2(t)}^T \approx N_0 W_n = N_p, \tag{7}$$

where N_0 is the single-sided power spectral density, W_n is the system bandwidth in Hz, and N_p is the detected noise power. Thus, the time-varying SNR is

$$\mathrm{SNR}(t) = \frac{\overline{x^2(t)}^T}{\overline{n^2(t)}^T} = \frac{\sum_{k=0}^{U-1} A_k^2(t)P_k}{N_0 W_n}. \tag{8}$$

2.2. Capacity Function

Define the short-time channel capacity $C(t)$ as $C(t) = W_n \log_2[1 + \mathrm{SNR}(t)]$, where W_n is the bandwidth of the spread-spectrum signal and SNR(t) is the time-varying SNR, which is the ratio of the mean-square value of the received signal to the mean-square value of noise. The capacity function $C(t)$ is

$$C(t) = W_n \log_2 \left[1 + \frac{\sum_{k=0}^{U-1} A_k^2(t) P_k}{N_0 W_n} \right] = W_n \log_2(e) \log_e \left[1 + \frac{\sum_{k=0}^{U-1} A_k^2(t) P_k}{N_0 W_n} \right]. \quad (9)$$

For sufficiently large W_n, (9) can be approximated as [16]

$$C(t) \approx \log_2(e) \frac{\sum_{k=0}^{U-1} A_k^2(t) P_k}{N_0}. \quad (10)$$

Hence, fluctuations in the channel capacity are proportional to the SNR variations [16]. This provides some justification in using SNR measurements in adapting to this communication channel.

3. ERROR PROBABILITY DISTRIBUTION AND DENSITY FUNCTIONS

To characterize the fading process in a communication system, error probability performance in a fading environment is used to calculate the probability of error averaged over the additive noise and signal-fading distributions. For slowly fading channels, it is useful to consider the probability of error over additive noise.

The probability of error, $P_e(t)$, is a random process, which depends on the fading of the signal. The probability of error can be expressed as a function of the energy-to-noise ratio [16]

$$P_e(t) = g\left[\frac{E(t)}{N_0}\right], \quad (11)$$

where $E(t)$ represents the detected signal energy variations due to signal fading. The function $g(x)$ depends on the modulation scheme used.

Substituting (8) into (11) (where SNR(t) is proportional to $\frac{E(t)}{N_0}$, the proportionality constant $\frac{1}{W_n}$), we have

$$P_e(t) = g\left[\frac{\sum_{k=0}^{U-1} A_k^2(t) P_k}{N_0 W_n}\right]. \quad (12)$$

3.1. Distribution Function

The distribution function of $P_e(t)$ is given by

$$\begin{aligned} F_{P_e(t)}(y) &= P[P_e(t) \le y] \\ &= P\left[g\left(\frac{E(t)}{N_0}\right) \le y\right] \\ &= P\left[\frac{E(t)}{N_0} \ge g^{-1}(y)\right] \\ &= 1 - F_{\frac{E(t)}{N_0}}\left[g^{-1}(y)\right], \end{aligned} \quad (13)$$

for $0 \le y \le \frac{1}{2}$, where $F_{E(t)/N_0}$ is the distribution function of the signal energy-to-noise variations. In (13), the inequality is reversed in the third step because $g(E(t)/N_0)$ is a decreasing function of $E(t)/N_0$.

Now, the detected signal energy-to-noise ratio, $E(t)/N_0$ is proportional to the SNR variations, which is in turn proportional to the square of the received signal amplitude. The proportionality factor is the transmitted bit energy-to-noise ratio, $R = E_b/N_0 = B^2 T/N_0$, where B represents the amplitude of the modulated signal, and T represents the bit period. Here, symbol duration is a bit period. In the case of coherent detection and automatic phase control (perfect carrier synchronization with the received modulated signal), we can write [16]

$$\frac{E(t)}{N_0} = R\frac{a^2(t)}{2}, \quad (14)$$

where $a(t)$ is the amplitude of the received signal which has a density function that is Rayleigh or Rician fading depending on the type of channel. It should be noted that $E(t)/N_0$ has an exponential distribution if $a(t)$ is Rayleigh distributed, and $E(t)/N_0$ has a chi-square (non-central) distribution if $a(t)$ is Rician distributed. If $a(t)$ is Rayleigh distributed, letting $Y \equiv E(t)/N_0$, we have

$$\begin{aligned} F_{\frac{E(t)}{N_0},\mathrm{RaF}}\left[g^{-1}(y)\right] &= F_{Y,\mathrm{RaF}}\left[g^{-1}(y)\right] \\ &= 1 - \exp\left[-\frac{g^{-1}(y)}{2\sigma^2}\right], \end{aligned} \quad (15)$$

where $g^{-1}(y) \geq 0$. From (14), (15) and [17], the mean value of the detected energy-to-noise variations, $\langle\frac{E(t)}{N_0}\rangle = 2\sigma^2 R$, if $a(t)$ is Rayleigh distributed. If $a(t)$ is Rician distributed,

$$F_{\frac{E(t)}{N_0},\mathrm{RiF}}\left[g^{-1}(y)\right] = F_{Y,\mathrm{RiF}}\left[g^{-1}(y)\right] = 1 - Q_1\left[\frac{s}{\sigma}, \frac{\sqrt{g^{-1}(y)}}{\sigma}\right], \quad (16)$$

where Q_1 is the Marcum's Q function [17]. From (14), (16) and [17], the mean value of the detected energy-to-noise variations, $\langle\frac{E(t)}{N_0}\rangle = (2\sigma^2 + 2\sigma^4\gamma^2)R$, if $a(t)$ is Rician distributed. Here, γ is the Rician fading parameter (ratio of the specular signal energy to the diffuse signal energy).

Substituting (15) into (13) and (16) into (13), we have

$$F_{P_e(t),\mathrm{RaF}}(y) = \exp\left[-\frac{g^{-1}(y)}{2\sigma^2}\right] \quad (17)$$

and

$$F_{P_e(t),\mathrm{RiF}}(y) = Q_1\left[\frac{s}{\sigma}, \frac{\sqrt{g^{-1}(y)}}{\sigma}\right], \quad (18)$$

respectively. Equations 17 and 18 represent the distribution function of the probability of error for Rayleigh and Rician fading channels, respectively.

3.2. Density Function

The density function of $P_e(t)$ (Rayleigh fading case) is given by

$$f_{P_e(t),\mathrm{RaF}}(y) = \frac{\partial}{\partial y}F_{P_e(t),\mathrm{RaF}}(y) = \frac{\partial}{\partial y}\left\{\exp\left[-\frac{g^{-1}(y)}{2\sigma^2}\right]\right\} = -\frac{1}{2\sigma^2}\exp\left[-\frac{g^{-1}(y)}{2\sigma^2}\right]\frac{\partial g^{-1}(y)}{\partial y}. \quad (19)$$

Since $g^{-1}(y)$ is a decreasing function of y, we have $\frac{\partial g^{-1}(y)}{\partial y} > 0$. Let $\frac{\partial g^{-1}(y)}{\partial y} = -c(y)$, where $c(y) > 0\ \forall y$. So, (19) can be rewritten as

$$f_{P_e(t),\mathrm{RaF}}(y) = \frac{c(y)}{2\sigma^2}\exp\left[-\frac{g^{-1}(y)}{2\sigma^2}\right]. \quad (20)$$

The exact expression for the density function of $P_e(t)$ (Rayleigh fading case) is given by (A.4) in the Appendix section of this paper.

The density function of the detected signal energy-to-noise variations can be computed by differentiating (15) with respect to y.

3.3. Fading Margin

Fading margin is defined to be the design allowance that provides sufficient "system gain" or "sensitivity" to accommodate the expected amount of fading for the purpose of ensuring the required quality of service [16]. It is also defined as the amount by which a received signal level may be reduced without causing system performance to fall below a specified value of threshold [1].

Fading margin depends on the distribution function, $F_{P_e(t)}(y)$, of the probability of error $P_e(t)$. It is needed to determine the mean signal energy-to-noise ratio so that $P_e(t) > \alpha$ with probability β, where α is the probability of error specification, and β is the probability of unsatisfactory operation, i.e.,

$$P\{P_e(t) > \alpha\} = \beta. \quad (21)$$

Equation (21) can be rewritten as

$$\beta = 1 - P\{P_e(t) \leq \alpha\} = 1 - F_{P_e(t),\mathrm{RaF}}(\alpha) = 1 - \exp\left[-\frac{g^{-1}(\alpha)}{2\sigma^2}\right] \quad (22)$$

if $a(t)$ is Rayleigh distributed, and

$$\beta = 1 - F_{P_e(t),\mathrm{RiF}}(\alpha) = 1 - Q_1\left(\frac{s}{\sigma}, \frac{\sqrt{g^{-1}(\alpha)}}{\sigma}\right) \quad (23)$$

if $a(t)$ is Rician distributed.

Rearranging (22), we have

$$g^{-1}(\alpha) = -2\sigma^2\log_e(1-\beta). \quad (24)$$

Equation 24 gives the SNR required to maintain a probability of error specification, α, for a Rayleigh fading channel.

Rearranging (23), we have

$$g^{-1}(\alpha) = z'_\beta, \quad (25)$$

where z'_β is the β percentage point of the distribution given in (16). Equation 25 gives the SNR required to maintain a probability of error specification, α, for a Rician fading channel.

In a non-adaptive system, the mean signal energy-to-noise ratio can be required so that $P_e(t)$ exceeds a prescribed value α with probability β [16]. The ratio of this mean to the signal energy-to-noise ratio $g^{-1}(\alpha)$ required in the absence of fading to achieve error probability α is referred to as "fading margin" [16].

Thus, the fading margin, M_{RaF}, in the case of Rayleigh fading is given by

$$M_{\mathrm{RaF}} = \frac{\langle \frac{E(t)}{N_0} \rangle_{\mathrm{RaF}}}{g^{-1}(\alpha)} = \frac{2\sigma^2}{g^{-1}(\alpha)} = -\frac{1}{\log_e(1-\beta)}, \quad \text{from (24)} \tag{26}$$

where $2\sigma^2$ is the mean value of the signal energy-to-noise variations due to Rayleigh fading. The fading margin, M_{RiF}, in the case of Rician fading is given by

$$M_{\mathrm{RiF}} = \frac{\langle \frac{E(t)}{N_0} \rangle_{\mathrm{RiF}}}{g^{-1}(\alpha)} = \frac{2\sigma^2 + 2\sigma^4\gamma^2}{g^{-1}(\alpha)} = \frac{2\sigma^2 + 2\sigma^4\gamma^2}{z'_\beta}, \quad \text{from(25)} \tag{27}$$

where $(2\sigma^2 + 2\sigma^4\gamma^2)$ is the mean value of the signal energy-to-noise variations due to Rician fading.

3.4. Theoretical Analysis

Figure 2 shows the fading margin as a function of the fading parameter γ for various values of β. The variance σ^2, of the Gaussian random variables is a constant (0.5) throughout the analysis. The variance of the AWGN noise is kept constant throughout the analysis. The fading margin for Rayleigh fading channels (denoted by $\gamma = 0$) is computed from (26). For Rician fading channels, the fading margin is given by (27). The quantity z'_β is the β percentage point of the distribution function given in (16), i.e., the value of z'_β is that value of $g^{-1}(y)$ for which the right hand side of (16) becomes equal to β.

Figure 2 shows that for Rayleigh fading channels, the system should provide the highest system gain (transmitted signal energy-to-noise ratio) to ensure the desired probability of error (quality of service). As the value of the fading parameter gradually increases, the fading margin decreases, indicating a decrease in the required system gain. As the probability of unsatisfactory operation increases, the relative system gain decreases for a specific value of the fading parameter γ.

Figure 3 shows the distribution function of the probability of error in a Rayleigh fading channel for various mean values of the detected signal energy-to-noise variations, $\langle \frac{E(t)}{N_0} \rangle$. We choose $R = 1$ in the discussion of distribution and density functions. For higher values of $\langle \frac{E(t)}{N_0} \rangle$, the distribution function reaches a maximum quicker than those with lower $\langle \frac{E(t)}{N_0} \rangle$. Thus, the peak value of the cumulative distribution function of the probability of error is higher for a system with larger $\langle \frac{E(t)}{N_0} \rangle$ as compared to a system with lower $\langle \frac{E(t)}{N_0} \rangle$. For higher values of $\langle \frac{E(t)}{N_0} \rangle$, the probability of achieving low probability of error is higher than the case with lower values of $\langle \frac{E(t)}{N_0} \rangle$.

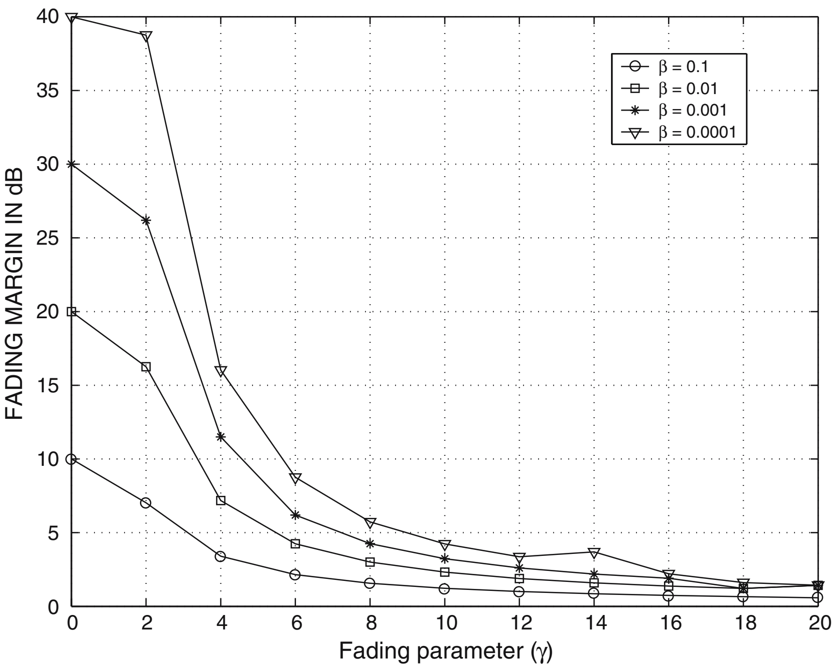

Fig. 2. Fading margin in decibels as a function of γ, the fading parameter for $\beta = 0.0001$, 0.001, 0.01, 0.1.

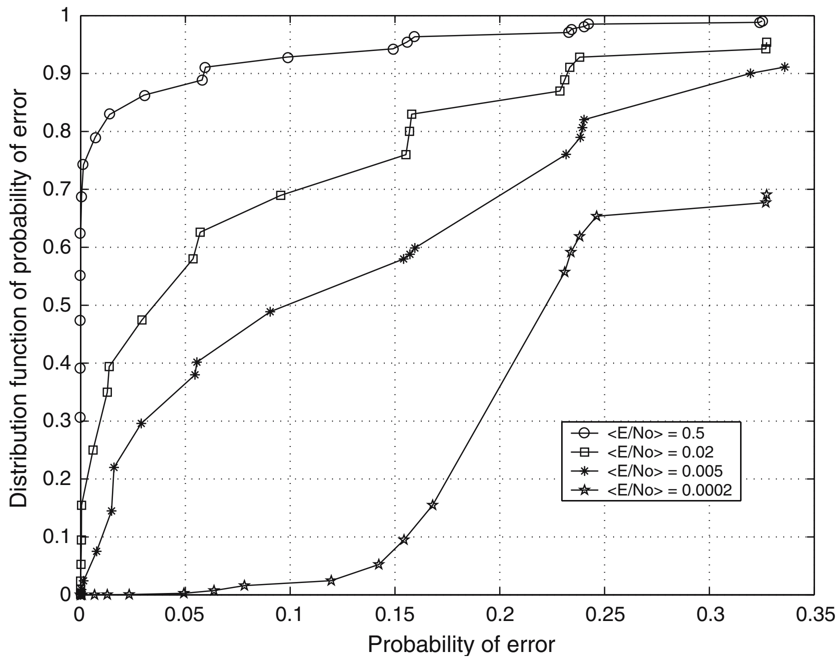

Fig. 3. Distribution function of the probability of error for a Rayleigh fading channel with $\langle\frac{E(t)}{N_0}\rangle = 0.5, 0.02, 0.005, 0.0002$.

The distribution function peaks up quicker for higher values of $\langle\frac{E(t)}{N_0}\rangle$ because it can find large number of probability values which have low probability of error as compared to the case when $\langle\frac{E(t)}{N_0}\rangle$ is lower.

Figure 4 shows the density function of the probability of error in a Rayleigh fading channel for various mean values of the detected signal energy-to-noise variations, $\langle\frac{E(t)}{N_0}\rangle$. The amplitude of the density function curves indicate the relative frequencies

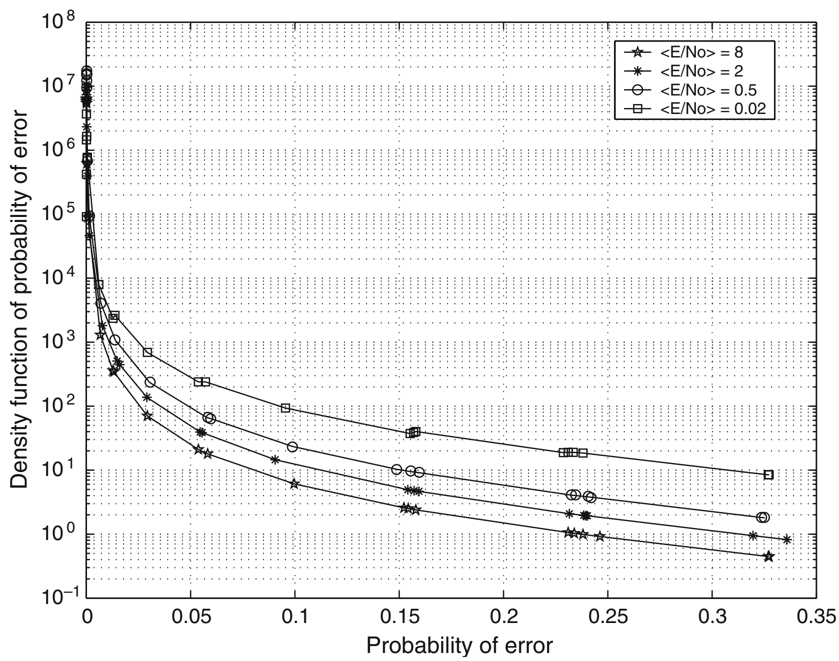

Fig. 4. Density function of the probability of error for a Rayleigh fading channel with $\langle\frac{E(t)}{N_0}\rangle = 8, 2, 0.5, 0.02$.

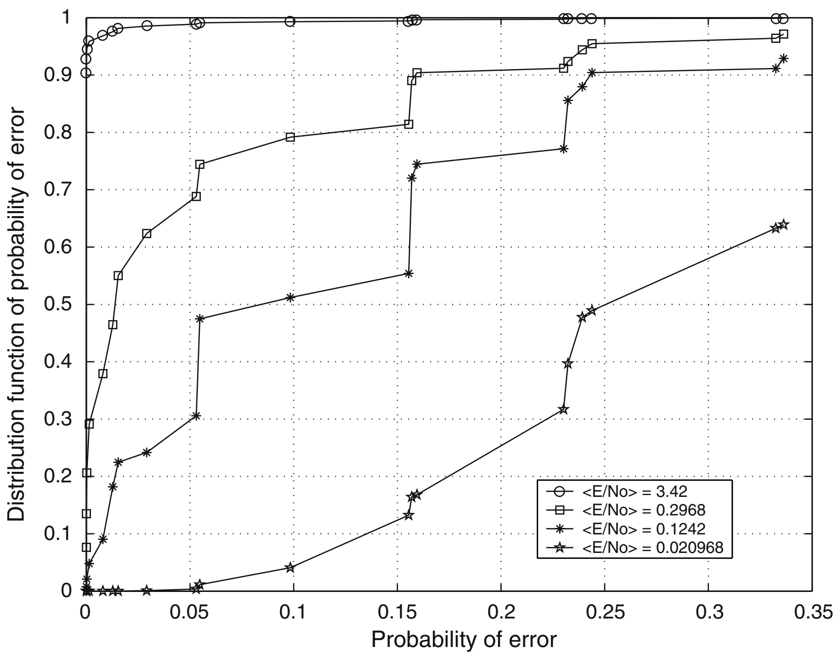

Fig. 5. Distribution function of the probability of error for a Rician fading channel with $\langle\frac{E(t)}{N_0}\rangle = 3.42, 0.2968, 0.1242, 0.020968, \gamma = 2.2$ (constant), and different variances.

of the random variable, $P_e(t)$, occurring in the range $0 < P_e(t) < 0.35$. The frequencies are higher for a lower $\langle\frac{E(t)}{N_0}\rangle$, and the frequencies are lower for a higher $\langle\frac{E(t)}{N_0}\rangle$, which is expected.

Figure 5 shows the distribution function of the probability of error in a Rician fading channel for various mean values of $\langle\frac{E(t)}{N_0}\rangle$. Again, for higher values of $\langle\frac{E(t)}{N_0}\rangle$, the distribution function reaches a maximum

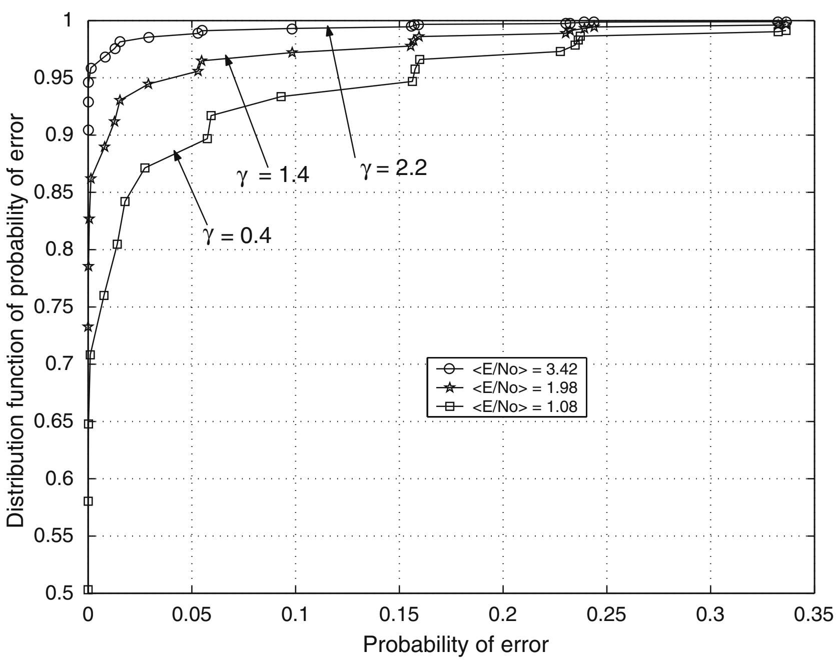

Fig. 6 Distribution function of the probability of error for a Rician fading channel with $\langle\frac{E(t)}{N_0}\rangle = 3.42, 1.98, 1.08, \sigma^2 = 0.5$ (constant), and $\gamma = 2.2$, 1.4, 0.4.

quicker than those with lower $\langle\frac{E(t)}{N_0}\rangle$. The sum of the probabilities of finding lower $P_e(t)$ is higher for a system with higher $\langle\frac{E(t)}{N_0}\rangle$ than for a system with lower $\langle\frac{E(t)}{N_0}\rangle$. In this analysis, the variance changes for each value of $\langle\frac{E(t)}{N_0}\rangle$.

Figure 6 shows the distribution function of the probability of error in a Rician fading channel for various mean values of $\langle\frac{E(t)}{N_0}\rangle$ with constant σ^2 and different γ. Higher values of γ correspond to higher values of $\langle\frac{E(t)}{N_0}\rangle$, which would mean that the distribution function reaches a maximum quicker than those with lower $\langle\frac{E(t)}{N_0}\rangle$.

4. CONCLUSIONS

This paper provides a justification on using SNR measurements in adapting the signal energy to increase channel capacity. The error probability distribution and density functions are introduced to characterize the fading process involved in an A-CDMA system. The probability distribution and density functions of the probability of error in an A-CDMA system for Rayleigh and Rician fading channels are derived and the results are plotted. Fading margin provides the base station with an idea over the range which the received signal needs to be varied to ensure the desired QoS. The base station controls the transmitter power constantly using the fading margin as a measure. The concept of fading margin is used to quantify the amount of system gain required to accommodate the expected level of fading in the system in order to ensure a desired probability of error. The fading margin is found to be largest for a Rayleigh fading channel.

APPENDIX

For a coherent BPSK system, the probability of bit error is given by [17]

$$y = g(\gamma_c) = \frac{1}{2}\left[1 - \sqrt{\frac{\gamma_c}{1+\gamma_c}}\right], \tag{A.1}$$

where γ_c is the signal-to-noise ratio. In our case, we can consider $\gamma_c \equiv$ SNR. So, $g^{-1}(y) \equiv$ SNR. Let $x \equiv \gamma_c$. Then $\frac{\partial g^{-1}(y)}{\partial y} = \frac{\partial x}{\partial y}$. From (A.1), we have

$$\gamma_c = \frac{(1-2y)^2}{4(y-y^2)}. \tag{A.2}$$

From (A.2), we have

$$\begin{aligned}\frac{\partial \mathrm{SNR}}{\partial y} &= \frac{\partial}{\partial y}\left\{\frac{(1-2y)^2}{4(y-y^2)}\right\}\\ &= \frac{1}{4}\left\{\frac{-4(y-3y^2+2y^3)-(1-8y^3-6y+12y^2)}{(y-y^2)^2}\right\}\\ &= \frac{1}{4}\left\{\frac{2y-1}{(y-y^2)^2}\right\} = -c(y)\end{aligned} \tag{A.3}$$

since SNR is a decreasing function of y. Substituting the result of (A.3) for $c(y)$ in (20), we have

$$f_{P_e(t),\mathrm{RaF}}(y) = \frac{1}{8\sigma^2}\exp\left[\frac{-g^{-1}(y)}{2\sigma^2}\right]\left[\frac{1-2y}{(y-y^2)^2}\right]. \tag{A.4}$$

Equation A.4 gives the density function of the probability of error for a Rayleigh fading channel.

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